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REFLECTION OF PLANE WAVES FROM A FREE SURFACE OF A GENERALIZED MAGNETO-THERMOELASTIC SOLID HALF-SPACE WITH DIFFUSION

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Green-Naghdi's theory of generalized thermoelasticity is applied to study the reflection of P and SV waves from the free surface of a magneto-thermoelastic solid half-space. The boundary conditions are satisfied by appropriate potential functions to obtain a system of four non-homogeneous equations in reflection coefficients. The reflection coefficients depend upon the angle of incidence of P and SV waves, magnetic field, thermal field, diffusion parameters and other material constants. The numerical values of the modulus of the reflection coefficients are shown graphically with the angle of incidence of P and SV waves. The effect of magnetic field is observed significantly on various reflected waves.

Keywords: generalized thermoelasticity, plane waves, reflection, magnetic field, diffusion

1. Introduction

Lord and Shulman (1967) developed a theory of generalized thermoelasticity by including a flux rate term into the Fourier law of heat conduction, which avoids the unrealistic phenomenon of the infinite speed of heat propagation in classical model given by Biot (1956). They obtained a hyperbolic heat transport equation which ensures the finite speed of thermal signals. Green and Lindsay (1972) formulated another theory of generalized thermoelasticity known as temperature rate dependent thermoelasticity with two relaxation times, which obeys classical Fourier's law of heat conduction and also admits a finite speed of heat propagation. Ignaczak and Ostoja-Starzewski (2009) presented a unified treatment of both Lord-Shulman and Green-Lindsay theories. Apart from these theories of generalized thermoelasticity, Green and Naghdi (1991, 1993) formulated a theory of generalized thermoelasticity by including "thermal displacement gradient" among independent constitutive variables, known as the theory of thermoelasticity without energy dissipation. Chandrasekharaiah (1986) considered this wave like thermal phenomenon as 'second sound'. In a review article, Hetnarski and Ignaczak (1999) presented these theories of generalized thermoelasticity.

Wave propagation in a generalized thermoelastic media with additional parameters like diffusion, magnetic field, anisotropy, porosity, viscosity, microstructure, temperature and other parameters provide vital information about the existence of new or modified waves. This information is useful for experimental seismologists in correcting earthquake estimation. Some relevant studies on wave propagation in generalized thermoelasticity are studied by various authors. Notable among them are Sinha and Sinha (1974), Sinha and Elsibai (1996), Sinha and Elsibai (1997), Abd-Alla and Al-dawy (2000), Sharma *et al.* (2003), Singh (2010), Singh and Yadav (2012) and Singh (2013).

Thermo-diffusion in an elastic solid is due to the fields of temperature, mass diffusion and that of strain. There is an exchange of heat and mass in the environment during the process of thermo-diffusion in an elastic solid. The thermo-diffusion phenomenon in solids is used to describe the process of thermo-mechanical treatment of metals. This phenomenon is of great concern due to its many geophysical and industrial applications. For example, oil companies are interested in the process of thermo-diffusion for more efficient extraction of oil from oil deposits. The thermo-diffusion phenomenon also finds its application in the field associated with the advent of semiconductor devices and the advancement of microelectronics (Oriani, 1969).

Nowacki (1974, 1976) developed the coupled theory of thermoelastic diffusion and studied some dynamical problems of diffusion in solids. Following Lord and Shulman (1967), Sherief *et al.* (2004) developed a theory of generalized thermoelastic diffusion, which allows finite speeds of propagation of waves. Singh (2005, 2006) studied the wave propagation in a thermoelastic solid with diffusion in context of Lord-Shulman and Green-Lindsay theories, and showed the existence of three Coupled Longitudinal waves and a SV wave in a two-dimensional model. Various other problems in elastic solids with thermo-diffusion were studied by many researchers, see Abo-Dahab and Singh (2009), Aoudai (2006, 2007, 2008), Choudhary and Deswal (2008), Deswal and Choudhary (2009), Othman *et al.* (2009), Singh (2013).

In the present paper, the Green and Naghdi theory of thermoelasticity without dissipation is followed to study the reflection from a stress-free surface of a magneto-thermoelastic solid half-space with diffusion. In Section 2, the basic equations for an isotropic, homogeneous generalized thermoelastic medium are formulated in the presence of diffusion and magnetic field. In Section 3, the basic equations are solved for plane wave solutions in the xz-plane to show the existence of three P waves and a SV wave. In Section 4, the reflection phenomenon of incident P and SV is considered. The appropriate boundary conditions are satisfied by appropriate potential functions to obtain the reflection coefficients of various reflected waves. A particular example of the model is chosen in Section 5 to compute the numerical values of the reflection coefficients against the angle of incidence for different values of the magnetic parameter. The effect of magnetic field on various reflected waves is depicted graphically.

2. Basic equations

Following Green and Naghdi (1993) and Sherief *et al.* (2004), the governing equations for an isotropic, homogenous generalized magneto-thermoelastic solid with diffusion at constant temperature T_0 in the absence of body forces are:

(i) The constitutive equations

$$\begin{aligned} &\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta_1 \Theta - \beta_2 C) \qquad && \rho T_0 S = \rho C_E \Theta + \beta_1 T_0 e_{kk} + a T_0 C \\ &P = -\beta_2 e_{kk} + bC - a\Theta \end{aligned} \tag{2.1}$$

(ii) Maxwell's stress equations

$$\Gamma_{ij} = \mu_e[H_i h_j + H_j h_i - \mathbf{H}\cdot\mathbf{h}\delta_{ij}] \tag{2.2}$$

Assuming that linearized Maxwell's equations are governing the electromagnetic field and the medium is a perfect electric conductor in the absence of displacement current, then

$$\operatorname{curl}\mathbf{h} = \mathbf{j} \qquad \operatorname{curl}\mathbf{E} = -\mu_e\frac{\partial\mathbf{h}}{\partial t} \qquad \operatorname{div}\mathbf{h} = 0 \qquad \operatorname{div}\mathbf{E} = 0 \tag{2.3}$$

where H is a constant primary magnetic field acting in the direction y, and

$$\mathbf{h} = \operatorname{curl}(\mathbf{u}\times\mathbf{H}_0) \qquad \mathbf{H} = \mathbf{H}_0 + \mathbf{h} \qquad \mathbf{H}_0 = [0, H, 0] \tag{2.4}$$

(iii) The equation of motion

$$\sigma_{ij,j} + \mathit{\Gamma}_{ji,j} = \rho\ddot{u}_i \tag{2.5}$$

(iv) The equation of heat conduction

$$K^*\Theta_{,ii} = \rho C_E\ddot{\Theta} + \beta_1 T_0 \ddot{e}_{kk} + aT_0\ddot{C} \tag{2.6}$$

(v) The equation of mass diffusion

$$D\beta_2 e_{kk,ii} + Da\Theta_{,ii} + \dot{C} - DbC_{,ii} = 0 \tag{2.7}$$

where ρ is density of the medium, λ, μ are Lame's constants, T is absolute temperature, T_0 is temperature of the medium in its natural state, $\Theta = T - T_0$ is the change in temperature such that $|\Theta/T_0| \ll 1$, σ_{ij} are components of the stress tensor, e_{ij} are components of the strain tensor, u_i are components of the displacement vector, S is entropy per unit mass, P is chemical potential per unit mass, C is mass concentration, C_E is specific heat at constant strain, D is thermo-diffusion constant, which ensures that the equation satisfied by the concentration C will also predict a finite speed of propagation of matter from one medium to another, a is a constant to measure the thermo-diffusion effects, b is a constant to measure the diffusive effects, δ_{ij} is the Kronecker delta, $\beta_1 = (3\lambda + 2\mu)\alpha_t$, $\beta_2 = (3\lambda + 2\mu)\alpha_c$ and $K^* = C_E(\lambda + 2\mu)/4$ are material constants, α_t is a coefficient of linear thermal expansion and α_c is a coefficient of linear diffusion expansion, $\mathbf{H}_0$ is the primary constant magnetic field, $\mathbf{h}$ is the perturbed magnetic field over $\mathbf{H}_0$.

3. Plane wave solution in the xz-plane

With the help of equations (2.1) to (2.7), the governing field equations for a homogeneous, isotropic generalized magneto-thermoelastic solid with diffusion in the xz-plane are written as

$$\begin{aligned}
&(\lambda + 2\mu)u_{1,11} + (\lambda + \mu)u_{3,13} + \mu u_{1,33} - \beta_1\Theta_{,1} - \beta_2 C_{,1} = \rho\ddot{u}_1\\
&(\lambda + 2\mu)u_{3,33} + (\lambda + \mu)u_{1,13} + \mu u_{3,11} - \beta_1\Theta_{,3} - \beta_2 C_{,3} = \rho\ddot{u}_3\\
&K^*\nabla^2\Theta = \rho C_E\ddot{\Theta} + \beta_1 T_0(\ddot{e}_{11} + \ddot{e}_{33}) + aT_0\ddot{C}\\
&D\beta_2\nabla^2 e + Da\nabla^2\Theta - Db\nabla^2 C + \dot{C} = 0
\end{aligned} \tag{3.1}$$

where $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial z^2)$.

Helmholtz's representations of the displacement components u_1 and u_3 in terms of scalar potential functions φ and ψ are

$$u_1 = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z} \qquad u_3 = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x} \tag{3.2}$$

Using equation (3.2) in equations (3.1), we obtain

$$\mu\nabla^2\psi = \rho\ddot{\psi} \tag{3.3}$$

and

$$\begin{aligned}
&(\lambda + 2\mu + \mu_e H_0^2)\nabla^2\varphi - \beta_1\Theta - \beta_2 C = \rho\ddot{\varphi}\\
&K^*\nabla^2\Theta = \rho C_E\ddot{\Theta} + \beta_1 T_0\frac{\partial}{\partial t}\nabla^2\varphi + aT_0\ddot{C}\\
&D\beta_2\nabla^4\varphi + Da\nabla^2\Theta - Db\nabla^2 C + \dot{C} = 0
\end{aligned} \tag{3.4}$$

Equation (3.3) is uncoupled and equations (3.4) are coupled in potential functions φ, Θ and C. From equations (3.4), it is noticed that the P wave is affected by thermal, diffusion and magnetic fields, and the SV wave remains unaffected. The solution to equation (3.3) suggests propagation of the SV wave with velocity $\sqrt{\mu/\rho}$.

The solutions to equations (3.4) are now sought in the form of the harmonic traveling wave

$$[\varphi, \Theta, C] = [\varphi_0, \Theta_0, C_0] \exp[\mathrm{i}k(x\sin\theta + z\cos\theta - vt)] \tag{3.5}$$

where $(\sin\theta, z\cos\theta)$ is the projection of the wave normal to the xz-plane, φ_0, Θ_0, C_0 are constants, k is the wave number and v is the phase speed.

By substituting equation (3.5) into equations (3.4), we obtain a homogenous system of equations in φ_0, Θ_0 and C_0 which has a non-trivial solution if ξ satisfies the following cubic equation

$$\xi^3 + L\xi^2 + M\xi + N = 0 \tag{3.6}$$

Here

$$\begin{aligned}
&\xi = \rho v^2 \qquad L = -[\varepsilon + \varepsilon\varepsilon_2\varepsilon_1^2 + d_1 + d_2 + (\lambda + 2\mu + \mu_e H_0^2)]\\
&M = (\lambda + 2\mu + \mu_e H_0^2)(d_1 + d_2 + \varepsilon\varepsilon_2\varepsilon_1^2) + d_1 d_2 + d_2\varepsilon - 2\varepsilon\varepsilon_1\varepsilon_2 - \varepsilon_2\\
&N = -d_1 d_2(\lambda + 2\mu + \mu_e H_0^2) + \varepsilon_2 d_1
\end{aligned}$$

where

$$\begin{aligned}
&d_1 = \frac{K^*}{C_E} \qquad && d_2 = \frac{\rho Db}{\tau} \qquad && \varepsilon = \frac{\beta_1^2 T_0}{\rho C_E}\\
&\varepsilon_1 = -\frac{a}{\beta_1\beta_2} \qquad && \varepsilon_2 = \frac{\rho D\beta_2^2}{\tau} \qquad && \tau = \frac{\mathrm{i}}{kv}
\end{aligned}$$

Cubic equation (3.6) can be solved with the help of Cardano's method. The three roots of this equation correspond to three plane longitudinal waves if v^2 is real and positive. Following Singh (2005, 2006), the three real roots v_1, v_2 and v_3, correspond to P1, P2 and P3 waves, where P1 and P2 waves are observed the fastest and slowest, respectively.

4. Reflection

For the incidence of P1 and SV waves, the boundary conditions at the free surface are satisfied if the incident P1 or SV wave gives rise to the reflected SV and three reflected coupled longitudinal waves (P1, P2 and P3). The complete geometry showing the incident and reflected waves from the free surface of a magneto-thermoelastic solid half-space with diffusion is shown in Fig. 1.

The appropriate displacement potential functions φ and ψ, temperature Θ and concentration C for the incident and reflected waves are

$$\begin{aligned}
\varphi &= A_0 \exp[\mathrm{i}k_1(x\sin\theta_0 + z\cos\theta_0) - \mathrm{i}\omega t] + \sum_{i=1}^{3} A_i \exp[\mathrm{i}k_i(x\sin\theta_i - z\cos\theta_i) - \mathrm{i}\omega t]\\
\Theta &= \varsigma_1 A_0 \exp[\mathrm{i}k_1(x\sin\theta_0 + z\cos\theta_0) - \mathrm{i}\omega t] + \sum_{i=1}^{3} \varsigma_i A_i \exp[\mathrm{i}k_i(x\sin\theta_i - z\cos\theta_i) - \mathrm{i}\omega t]\\
C &= \eta_1 A_0 \exp[\mathrm{i}k_1(x\sin\theta_0 + z\cos\theta_0) - \mathrm{i}\omega t] + \sum_{i=1}^{3} \eta_i A_i \exp[\mathrm{i}k_i(x\sin\theta_i - z\cos\theta_i) - \mathrm{i}\omega t]\\
\psi &= B_0 \exp[\mathrm{i}k_4(x\sin\theta_0 + z\cos\theta_0) - \mathrm{i}\omega t] + B_1 \exp[\mathrm{i}k_4(x\sin\theta_4 - z\cos\theta_4) - \mathrm{i}\omega t]
\end{aligned} \tag{4.1}$$

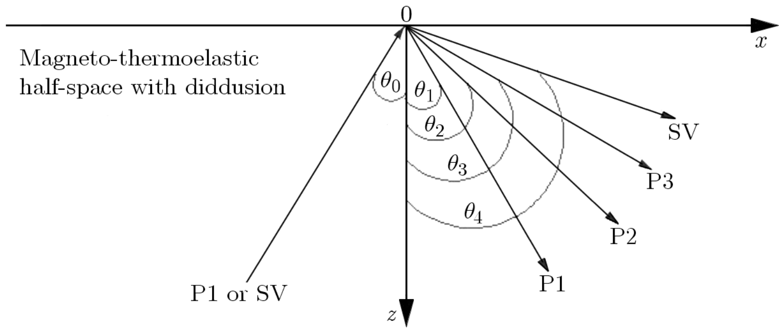

Fig. 1. Geometry of the problem showing incident and reflected waves

where the incident P1 or SV wave makes the angle θ_0 with the negative direction of the z-axis and reflected P1, P2, P3, and SV waves makes the angles θ_1, θ_2, θ_3 and θ_4, respectively, and for $i = 1,2,3$

$$\varsigma_i = \frac{k_i^2}{\beta_1} G_i(\lambda + 2\mu + \mu_e H_0^2 - \rho v_i^2) \qquad \eta_i = \frac{k_i^2}{\beta_1} F_i(\lambda + 2\mu + \mu_e H_0^2 - \rho v_i^2) \tag{4.2}$$

where

$$\begin{aligned} G_i &= \frac{\varepsilon\rho v_i^2(\varepsilon_1\varepsilon_2 - d_2 + \rho v_i^2)}{d_1\varepsilon_2 + \rho v_i^2[\varepsilon(d_2 - \rho v_i^2) - \varepsilon_2 - 2\varepsilon\varepsilon_1\varepsilon_2]} \\ F_i &= \frac{\varepsilon_2[\rho v_i^2(\varepsilon\varepsilon_1 + 1) - d_1]}{d_1\varepsilon_2 + \rho v_i^2[\varepsilon(d_2 - \rho v_i^2) - \varepsilon_2 - 2\varepsilon\varepsilon_1\varepsilon_2]} \end{aligned} \tag{4.3}$$

The required boundary conditions at the free surface $z = 0$ are the vanishing normal stresses, tangential stresses, heat flux and mass flux, i.e.

$$\sigma_{zz} + \varGamma_{zz} = 0 \qquad \sigma_{zx} + \varGamma_{zx} = 0 \qquad \frac{\partial \varTheta}{\partial z} = 0 \qquad \frac{\partial P}{\partial z} = 0 \qquad \text{on} \qquad z = 0 \tag{4.4}$$

The ratios of amplitudes of the reflected waves to the amplitude of the incident P1 wave, namely, A_1/A_0, A_2/A_0, A_3/A_0, B_1/A_0 give the reflection coefficients for reflected P1, reflected P2, reflected P3, and reflected SV waves, respectively. Similarly, for the incident SV wave A_1/B_0, A_2/B_0, A_3/B_0, B_1/B_0, are the reflection coefficients of P1, P2, P3 and SV waves, respectively. The wave numbers k_1, k_2, k_3, k_4 are connected by the angles of incidence and reflection as

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 = k_4 \sin\theta_4 \tag{4.5}$$

In order to satisfy the boundary conditions, relation (4.5) is also written as

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} = \frac{\sin\theta_4}{v_4} \tag{4.6}$$

where $v_4 = \sqrt{\mu/\rho}$ is the velocity of the SV wave and v_i $(i = 1,2,3)$ are the velocities of P1, P2 and P3 waves.

With the help of equations (2.1), (3.2) and (4.1), boundary condition (4.4) leads to a non-homogeneous system of four equations as

$$\sum_{j=1}^{4} a_{ij} Z_j = b_i \qquad i = 1,2,3,4 \tag{4.7}$$

where:
— for the incident P1-wave $\theta_0 = \theta_1$ and $j = 1, 2, 3$

$$a_{1j} = -(2\mu\mathcal{D}_{1j} + \lambda + \mu_e H_0^2 + G_j\mathcal{E}_j + F_j\mathcal{E}_j)\Big(\frac{v_1}{v_j}\Big)^2 \qquad a_{14} = 2\mu\sin\theta_0\sqrt{1-\sin^2\theta_0\Big(\frac{v_4}{v_1}\Big)^2}\frac{v_1}{v_4}$$

$$a_{2j} = 2\sin\theta_0\sqrt{\mathcal{D}_{1j}}\frac{v_1}{v_j} \qquad a_{24} = \Big[1-2\sin^2\theta_0\Big(\frac{v_4}{v_1}\Big)^2\Big]\Big(\frac{v_1}{v_4}\Big)^2$$

$$a_{3j} = \sqrt{\mathcal{D}_{1j}}\frac{G_i}{\beta_1}\mathcal{E}_j\Big(\frac{v_1}{v_j}\Big)^3 \qquad a_{34} = 0$$

$$a_{4j} = \sqrt{\mathcal{D}_{1j}}\Big[\beta_2 - a\frac{G_j}{\beta_1}\mathcal{E}_j + b\frac{F_j}{\beta_2}\mathcal{E}_j\Big(\frac{v_1}{v_j}\Big)^3\Big] \qquad a_{44} = 0$$

$$Z_1 = \frac{A_1}{A_0} \qquad Z_2 = \frac{A_2}{A_0} \qquad Z_3 = \frac{A_3}{A_0} \qquad Z_4 = \frac{B_1}{A_0}$$

$$b_1 = -a_{11} \qquad b_2 = a_{21} \qquad b_3 = a_{31} \qquad b_4 = a_{41}$$

where

$$\mathcal{D}_{1j} = 1-\sin^2\theta_0\Big(\frac{v_j}{v_1}\Big)^2 \qquad \mathcal{E}_j = \lambda + 2\mu + \mu_e H_0^2 - \rho v_j^2$$

— for incident SV wave $\theta_0 = \theta_4$ and $j = 1, 2, 3$

$$a_{1j} = -[2\mu\mathcal{D}_{4j} + \lambda + \mu_e H_0^2 + G_j\mathcal{E}_j + F_j\mathcal{E}_j]\Big(\frac{v_4}{v_j}\Big)^2 \qquad a_{14} = \mu\sin 2\theta_0$$

$$a_{2j} = 2\sin\theta_0\sqrt{\mathcal{D}_{4j}}\frac{v_4}{v_j} \qquad a_{24} = 1-2\sin^2\theta_0$$

$$a_{3j} = \sqrt{\mathcal{D}_{4j}}\frac{G_j}{\beta_1}\mathcal{E}_j\Big(\frac{v_1}{v_j}\Big)^3 \qquad a_{34} = 0$$

$$a_{4j} = \sqrt{\mathcal{D}_{4j}}\Big[\beta_2 - a\frac{G_j}{\beta_1}\mathcal{E}_j + b\frac{F_j}{\beta_2}\mathcal{E}_j\Big(\frac{v_4}{v_j}\Big)^3\Big] \qquad a_{44} = 0$$

$$Z_1 = \frac{A_1}{B_0} \qquad Z_2 = \frac{A_2}{B_0} \qquad Z_3 = \frac{A_3}{B_0} \qquad Z_4 = \frac{B_1}{B_0}$$

$$b_1 = a_{14} \qquad b_2 = -a_{24} \qquad b_3 = a_{34} \qquad b_4 = -a_{44}$$

where

$$\mathcal{D}_{4j} = 1-\sin^2\theta_0\Big(\frac{v_j}{v_4}\Big)^2$$

5. Application to a particular model

To study the numerical dependence of the reflection coefficients on various magnetic, thermal, diffusion, and other material constants, a particular example is chosen with the following physical constants at $T_0 = 300\,\mathrm{K}$: $\lambda = 5.775\cdot 10^{10}\,\mathrm{N/m^2}$, $\mu = 2.646\cdot 10^{10}\,\mathrm{N/m^2}$, $\rho = 2700\,\mathrm{kg/m^3}$, $C_E = 1.415\cdot 10^4\,\mathrm{J/(kg\,K)}$, $K = 3.223\cdot 10^3\,\mathrm{W/(m\,K)}$, $\tau = 0.04\,\mathrm{s}$, $\alpha_t = 0.137\cdot 10^2\,/\mathrm{K}$, $\alpha_c = 0.06\cdot 10^{-3}\,\mathrm{m^3/kg}$, $a = 0.137\cdot 10^{-3}\,\mathrm{m^2/(s^2K)}$, $b = 0.05\cdot 10^{-7}\,\mathrm{m^5/(kg\,s^2)}$, $D = 0.5\cdot 10^3\,\mathrm{kg\,s/m^3}$, $\omega = 20\,\mathrm{Hz}$.

Non-homogeneous system (4.7) of four simultaneous equations is solved by a Fortran program with the Gauss elimination method. Here, we concentrate only on observing the effects of magnetic field on the reflection coefficients, as the diffusion and relaxation effects were already shown by Singh (2005, 2006) in his papers on L-S and G-L theories.

The reflection coefficients of various reflected waves are computed for the range $0 < \theta_0 < 90°$ of the angle of incidence of P1 and SV waves when $H = 0$, 80 and $800\,\mathrm{A/m^2}$. The variations of these reflection coefficients against the angle of incidence are shown graphically in Figs. 2 and 3. For the incidence of P1 wave, the variations of reflection coefficients of various reflected waves against the angle of incidence are shown graphically in Fig. 2. The reflection coefficient of the SV wave is zero near the normal and grazing incidence. As the angle of incidence varies from normal to grazing incidence, it first increases to its maximum value and then decreases sharply as shown by the solid curve. If we compare the solid curve with other dotted curves, it is observed that the reflection coefficients of the SV wave fall sharply with the increase in value of the magnetic field at each angle of incidence except for the normal and grazing incidence. The reflection coefficient of the P1 wave first decreases slowly from its maximum value at the normal incidence and its starts increasing at angles near to the grazing incidence as shown by the solid curve. The comparison of the solid curve with other dotted curves shows the effect of magnetic field on the reflection coefficient of the P1 wave. Similarly, if we see the graphs of P2 and P3 waves, it is observed that the reflection coefficients of these waves fall sharply with an increase in the magnetic field at each angle of incidence except for the grazing incidence. The effect of magnetic field is maximum at the normal incidence, however, it decreases as the angle of incidence varies from the normal to grazing incidence, and then there is no effect of the magnetic field at the grazing incidence.

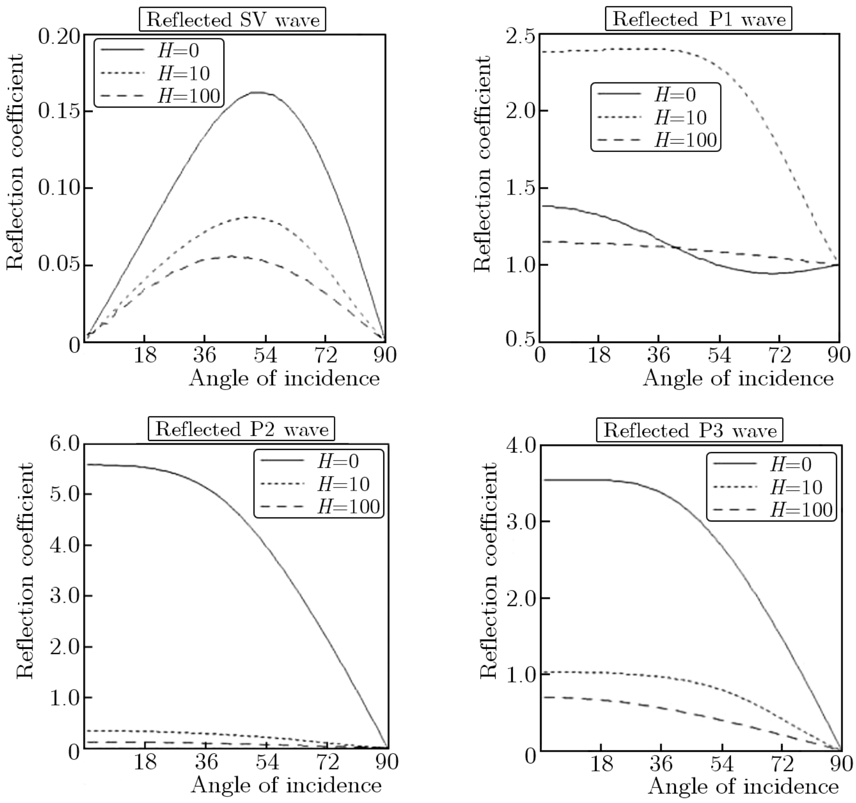

Fig. 2. Variations of reflection coefficients of reflected waves against the angle of incidence of the P1 wave

For the incidence of the SV wave, the variations of reflection coefficients of various reflected waves against the angle of incidence are shown graphically in Fig. 3. If we look at the four plots of reflection coefficients of the reflected SV, P1, P2 and P3 waves in Fig. 3, the effects of magnetic field are clearly observed at each angle of incidence, except for the normal incidence, grazing incidence and at the angle $45°$. The angle $45°$ of incidence of the SV wave is observed as the critical angle. The variations over the angle $45°$ will not appear if we compute the real parts of the coefficients only.

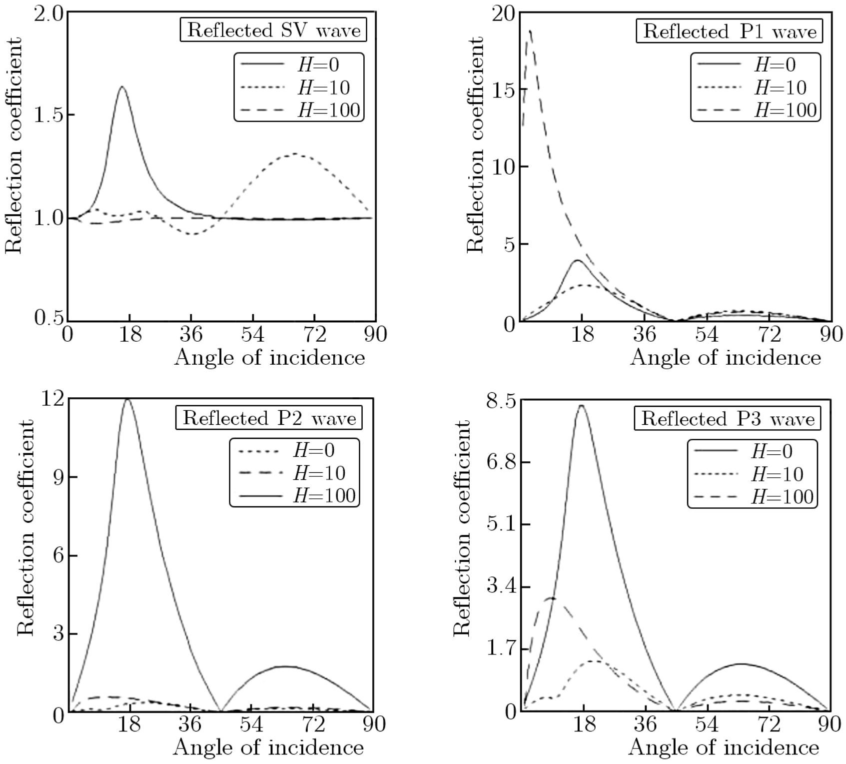

Fig. 3. Variations of reflection coefficients of reflected waves with the angle of incidence of the SV wave

6. Conclusions

The governing equations of a thermoelastic half-space with diffusion and magnetic field are formulated in context of G-N theory and are solved in a two-dimensional model. Similar to Singh (2005, 2006), there also exists three coupled longitudinal waves and a shear wave in the magneto-thermoelastic half-space with diffusion under G-N theory. These waves are considered for reflection from a thermally insulated half-space to obtain a non-homogeneous system of four equations in reflection coefficients. The numerical example shows that the presence of a magnetic field significantly changes the reflection coefficients of reflected waves for the incidence of both P and SV waves.

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